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Design and Implementation of Butterworth Filter

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ABSTRACT:Filters have become a very essential part in the digital signal processing field emerging exponentially in today's time. The function of a filter is to remove unwanted parts of the signal such as random noise that is also undesirable or to extract useful parts of the signal such as the components lying within a certain frequency range. In low pass filter, the lower frequencies will be passed without any attenuation and the higher frequencies which are the frequencies higher than the cut off frequency will be blocked or attenuated. In this paper, second and third Butterworth low pass filters have been discussed theoretically and experimentally.

KEYWORDS:Digital Signal Processing, Low Pass Filter, Butterworth Filter.

I.INTRODUCTION

Filter is a system that allows certain frequency to pass to its output and reject all other signals. Filters can be classified in many ways. A filter's frequency selectivity is probably the most common method of classification; filters can be classified according to the range of signal frequencies as follows; Low-pass, High-pass, Band-pass, and Band-stop (band-reject). In low pass filter, the lower frequencies will be passed without any attenuation and the higher frequencies which are the frequencies higher than the cut off frequency will be blocked or attenuated, this attenuation relies on the order of the filter. Fig 1 gives the basic idea behind each filter[1].

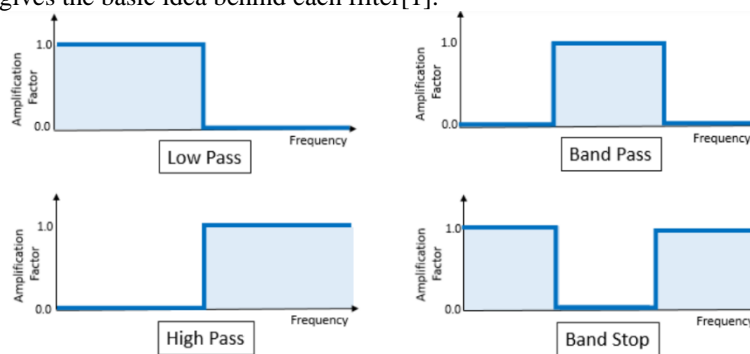


Fig. 1 Types of filters.

Filters can be classified based on the frequency response as: Butterworth, Chebychev I, Chebychev II, Elliptic filter. Since this paper deals with Butterworth filter, it might be worth to focus on this type in this overview. The Butterworth filter is a type of signal processing filter designed to have a frequency response as flat as possible in the passband. It is also referred as a maximally flat magnitude filter. It was first described in 1930 by the British engineer and physicist Stephen Butterworth in his paper entitled "On the Theory of Filter Amplifiers"[2]. Butterworth filters are one of the most used digital filters in motion analysis. They are fast and simple to use. This simplicity combined with a level of performance that is more than adequate for many applications means that the Butterworth filter is widely used in many areas of electronics from radio frequency (RF) to audio active filters. Since they are frequency-based, the effect of filtering can be easily understood and predicted [3].

The flatness of the output response increases as the order of the filter increases. Higher flatness corresponds to high precise filter. In practical implementation, there is no ideal filter. The gain and normalized response of the Butterworth filter for different orders are given in fig 2.

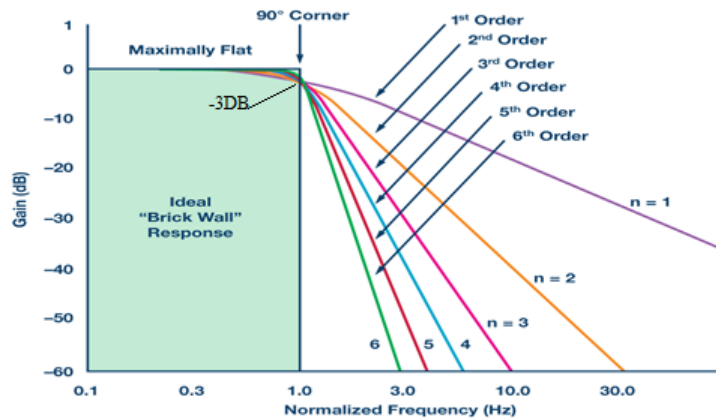


Fig. 2 Plot of the gain of Butterworth low-pass filters of orders 1 through 6.

Bilinear Z-transformation is the most common method for converting the transfer function $H(s)$ of the analog filter to the transfer function $H(z)$ of the digital filter by substituting the following expression: $S = (z - 1)/(z + 1)$. The magnitude response of low pass filter can be plotted based on this formula:

$$M_{lp} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$$

Where f_c is cut off frequency, n is the order of the filter.

Therefore, when $f = f_c$, the magnitude response is equal to 0.707 and so confirming the -3dB bandwidth, frequencies higher than f_c will be attenuated; very high frequencies will almost be stopped as for high values of (f) the amplitude response of the filter will be close to zero. Also, higher value of (n) leads to make the output response goes to zero sharper. Accordingly, as mentioned above increases in the order of the filter increases the flatness which increases the accuracy of the filter. In this paper, the bilinear transform will be applied to second and third order Butterworth low-pass filters. A mixture of sine waves was selected to demonstrate and test the proposed filter [4][5].

II. DESIGN OF SECOND AND THIED ORDER BUTTERWORTH LOW-PASS FILTER

2.1 SECOND ORDER BUTTERWORTH LOW-PASS FILTER

In this section the theoretical part of the second order filter is demonstrated. In order to design an accurate filter, number of Assumptions applied; sampling frequency is 4 KHz, and cut-off frequency is 500 Hz. Calculating the pre-warp frequency can be obtained through this equation:

$$\omega_{cp} = \tan\left(\frac{\pi * f_c}{f_s}\right) = \tan\left(\frac{180 * 500}{4000}\right) = 0.4142.$$

The standard S-domain transfer function for a 2nd order Butterworth filter is:

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \text{ By letting } \left(s = \frac{S}{\omega_{cp}}\right), H(s) = \frac{1}{\left(\frac{S}{\omega_{cp}}\right)^2 + \sqrt{2}\left(\frac{S}{\omega_{cp}}\right) + 1} = \frac{1}{\left(\frac{S}{0.4142}\right)^2 + \sqrt{2}\left(\frac{S}{0.4142}\right) + 1}$$

$$H(s) = \frac{0.4142^2}{s^2 + 0.5857 * s + 0.4142^2}$$

$$P_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\pm \sqrt{0.5857^2 - 4 * 0.1713}}{2}$$

$$P_1 = -0.292 + j0.2929$$

$$P_2 = -0.292 - j0.2929$$

Now, Transforming from the S domain to Z domain.

$$H(s) = \frac{0.1715}{s^2 + 0.5857 * s + 0.1715} \text{ By letting } \left(s = \frac{z - 1}{z + 1}\right), H(z) = \frac{0.1715}{\left(\frac{z-1}{z+1}\right)^2 + 0.5857 * \left(\frac{z-1}{z+1}\right) + 0.1715}$$

$$H(z) = \frac{0.17135 z^2 + 0.3427 z + 0.17135}{1.757z^2 - 1.6573z + 0.58565}$$

The poles and zeros of the filter can be determined using the same previous equation:

$$P_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1.6573) \pm \sqrt{(-1.6573)^2 - 4 * 1.757 * 0.58565}}{2 * 1.757}$$

$$P_1 = 0.471 + j 0.333$$

$$P_2 = 0.471 - j 0.333$$

Since the numerator is $0.1713 (z + 1)^2$, it easily can observe that the zeros are at:

$$Z_1 = -1 + j 0$$

$$Z_2 = -1 + j 0$$

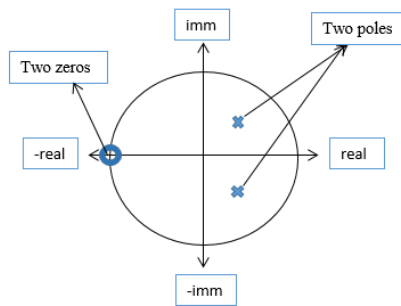


Fig. 3 The poles and zeros of the filter.

The difference equation implementation of the filter:

$$\frac{Y(Z)}{X(Z)} = \frac{0.17135 z^2 + 0.3427 z + 0.17135}{1.757z^2 - 1.6573z + 0.58565}$$

$$\frac{Y(Z)}{X(Z)} = \frac{0.17135 z^2 + 0.3427 z + 0.17135}{1.757z^2 - 1.6573z + 0.58565} * \frac{z^{-2}}{z^{-2}} = \frac{0.17135 + 0.3427 z^{-1} + 0.17135 z^{-2}}{1.757 - 1.6573z^{-1} + 0.58565 z^{-2}}$$

$$1.757 Y(Z) - 1.6573 Y(Z)z^{-1} + 0.58565 Y(Z)z^{-2} = 0.17135 X(Z) + 0.3427 X(Z)z^{-1} + 0.17135 X(Z) z^{-2}$$

By applying Z-inverse transform, the last equation become:

$$1.757 y(n) - 1.6573 y(n-1) + 0.58565 y(n-2) = 0.17135 x(n) + 0.3427 x(n-1) + 0.17135 x(n-2)$$

$$y(n) = 0.9435 y(n-1) - 0.3333 y(n-2) + 0.0975 x(n) + 0.195 x(n-1) + 0.0975 x(n-2)$$

The filter can be realised as in fig 4.

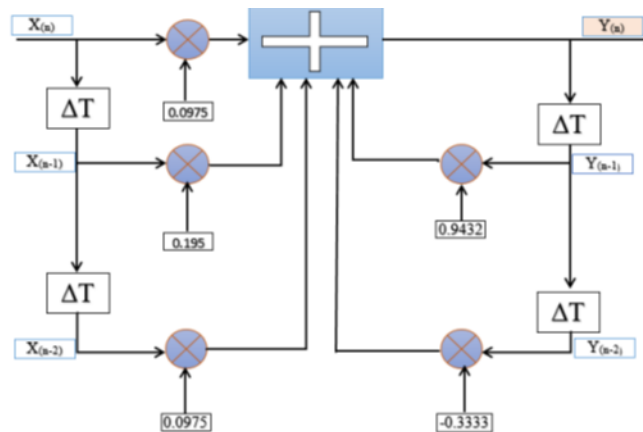


Fig.4 Filter configuration.



2.2THIRD ORDER BUTTERWORTH LOW-PASS FILTER

By applying the Assumption proposed for the second order filter, the design of third order Butterworth low-pass filter can be designed as follows; the sampling frequency is 4 KHz and cut-off frequency of 500 Hz. Calculating the pre-warp frequency can be obtained through this equation:

$$\omega_{cp} = \tan(\pi * f_c / f_s) = \tan(180 * 500 / 4000) = 0.4142.$$

The standard S-domain transfer function for a 3rd order Butterworth filter is:

$$H(s) = \frac{1}{(S + 1)(S^2 + S + 1)} \text{ By letting } \left(S = \frac{S}{\omega_{cp}} \right). H(s) = \frac{1}{\left(\left(\frac{S}{\omega_{cp}} \right) + 1 \right) \left(\left(\frac{S}{\omega_{cp}} \right)^2 + \left(\frac{S}{\omega_{cp}} \right) + 1 \right)}$$

$$H(s) = \frac{1}{\left(\frac{S+0.414}{0.414} \right) \left(\left(\frac{S}{0.4142} \right)^2 + \left(\frac{S}{0.4142} \right) + 1 \right)}$$

$$H(s) = \frac{0.414^3}{(S + 0.414)(S^2 + 0.414 * S + 0.414^2)}$$

Since there is no variable (s) in the numerator, there is no zeros.

It is obvious that the first pole is at: $P_1 = -0.414 + j0$

Finding the other two poles using the same procedure mentioned earlier:

$$P_2 = -0.207 + j 0.358$$

$$P_3 = -0.207 - j 0.358$$

Transforming from the S domain to Z domain, Working out the digital filter response in the Z domain.

$$H(s) = \frac{(0.414)^3}{(S + 0.414)(S^2 + 0.414 * S + (0.414)^2)} \text{ By letting } \left(S = \frac{z - 1}{z + 1} \right)$$

$$H(z) = \frac{(0.414)^3}{\left(\left(\frac{z-1}{z+1} \right) + 0.414 \right) \left(\left(\frac{z-1}{z+1} \right)^2 + 0.414 * \left(\frac{z-1}{z+1} \right) + 0.1715 \right)}$$

$$H(z) = \frac{(0.414)^3 (z + 1)^3}{(1.414z - 0.586)(1.585 z^2 - 1.658 z + 0.757)}$$

Now, finding the poles and the zeros of the filter:

Since the numerator is $0.709(z + 1)^3$, it easily can observe that the zeros are at:

$$Z_1 = -1 + j 0$$

$$Z_2 = -1 + j 0$$

$$Z_3 = -1 + j 0$$

Regarding to the poles the first pole P_1 is at $\frac{0.586}{1.414} = 0.414$

$$P_1 = 0.414 + j0, P_2 = 0.0523 + j0.45179, \text{ and } P_3 = 0.523 - j0.45179$$

The difference equation implementation of the filter:

$$\frac{Y(Z)}{X(Z)} = \frac{0.0709 + 0.212 z^{-1} + 0.212 z^{-2} + 0.0709 z^{-3}}{2.241 - 3.2732 z^{-1} + 2.04188 z^{-2} - 0.443 z^{-3}}$$

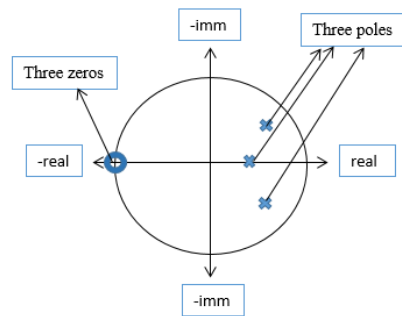


Fig. 5 The poles and zeros of the filter.

$$2.241 Y(Z) - 3.2732 Y(Z)z^{-1} + 2.04188 Y(Z)z^{-2} - 0.443 Y(Z)z^{-3} = 0.0709 X(Z) + 0.212 X(Z)z^{-1} + 0.212 X(Z)z^{-2} + 0.0709 X(Z)z^{-3}$$

By applying Z-inverse transform, the last equation become:

$$y(n) = 1.46 y(n-1) - 0.91 y(n-2) + 0.19 y(n-3) + 0.0316 x(n) + 0.094 x(n-1) + 0.0945 x(n-2) + 0.0316 x(n-3)$$

This Third order filter can be realised as shown in fig 6.

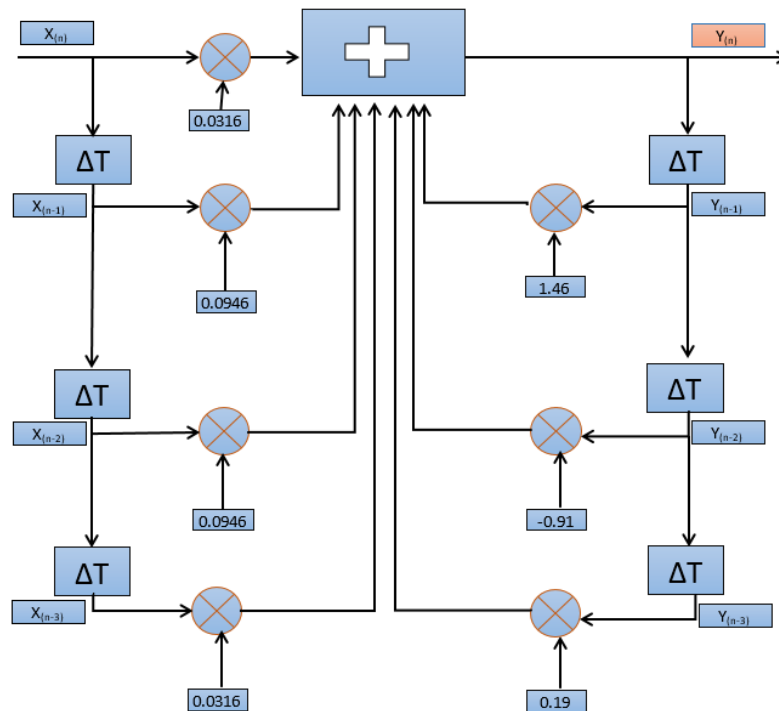


Fig.6 Filter configuration.

III. IMPLEMENTING AND TESTING THE FILTERS IN COMPUTER PROGRAM:

In this part section, C++ program has been developed to explain how these filters work and to what extent they can work effectively. In order to test these filters three sine waves at different frequencies are generated and mixed. After adding them together, they are passed through the proposed filter. The frequency of each sine wave is selected in a way that show how they will be affected after applying the filters. Therefore, as the cut off frequency is at 500Hz, this frequency is selected for one sine waves, the other waves will be at 200Hz and 1000Hz. The point behind selecting 1000Hz is due to the difference between the second and the third order filter at this frequency can be easily noticed. Frequency response are demonstrated and analysed before and after the 2nd & 3rd order Butterworth filters are applied.



Firstly: generating the three sine waves and add them in one signal. Sin wave at 200 Hz, 500Hz, and 1000Hz have been generated in the C++ code and are shown in the figures 7, 8, and 9. One point should be highlighted which is generating a sin wave in C++ is by this formula:

$$Signal = A \sin\left(\frac{3.14 * n}{T_n}\right)$$

Where, A is the amplitude of the signal, n: integer number (for the FOR loop) represent the number of samples, and T_n : is equal to $(\frac{F_s}{F})$. Where, F_s : Sampling frequency, and F is the frequency of the signal.

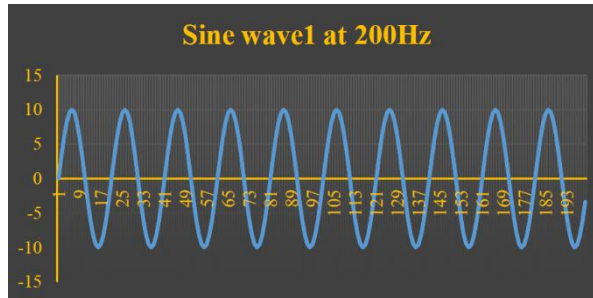


Fig. 7 Sin wave signal at 200Hz.

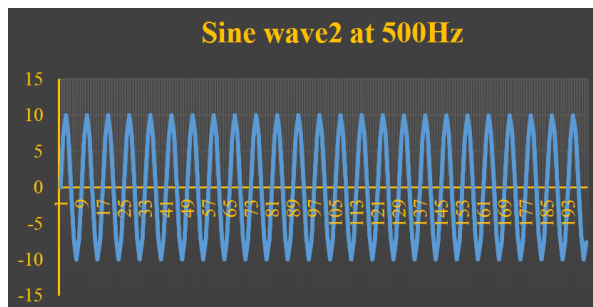


Fig. 8 Sin wave signal at 500Hz.

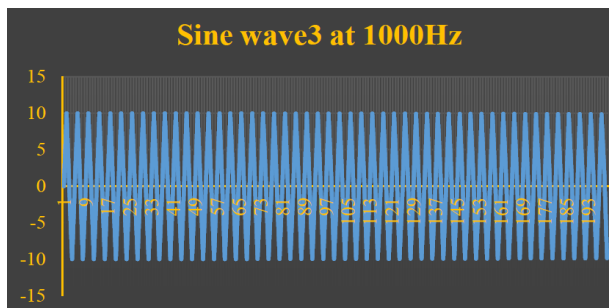


Fig. 9 Sin wave signal at 1000Hz.

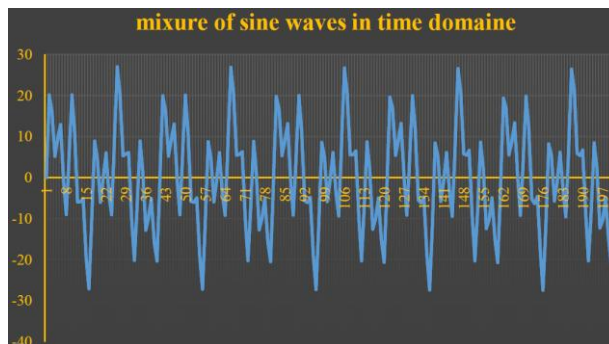


Fig. 10 Mixture of three sine waves in time domain.

The above three signals are added together to form a mixture of sine waves. The resulted signal is shown fig 10. As can be observed It is not easy to understand the constituent of this signal from this graph in the time domain. In order to see the different constituents of the signal, discrete Fourier transform (DFT) of the signal was taken which allows the designer to distinguish the different frequencies that form this signal.

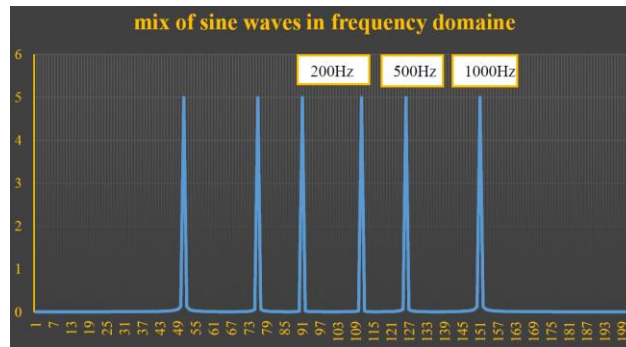


Fig. 11 Mixture of three sine waves in frequency domain.

It can be clearly observed from fig 11, the mixture signal constitutes of three signals at different frequencies. The DFT is a double-sided transform it shows two peaks represent each signal one on the right of the graph and the other on the left of the graph.

3.1 IMPLEMENTATION OF THE SECOND ORDER BUTTERWORTH FILTER:

In this section, a filter that has a cut off frequency of 500 Hz and a sampling frequency of 4 kHz is tested. Theoretically, any signal that is higher than 500 Hz is either attenuated or blocked, and frequencies less than 500Hz pass without any effect. The mixture of three sin waves is passed through the filter and the response is observed. Time domain and frequency domain of the resultant output of the filter is shown in fig 12 and 13, respectively.

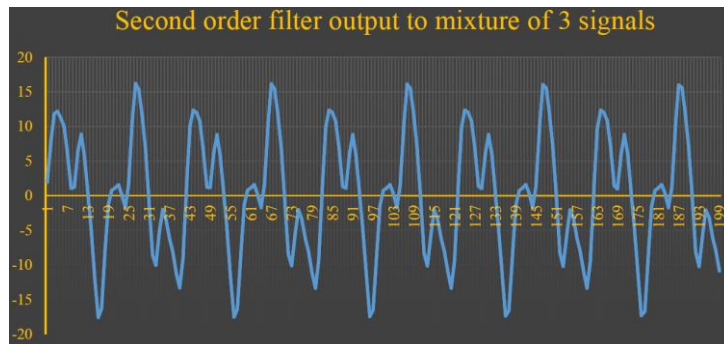


Fig. 12 The filtered signal in time domain.

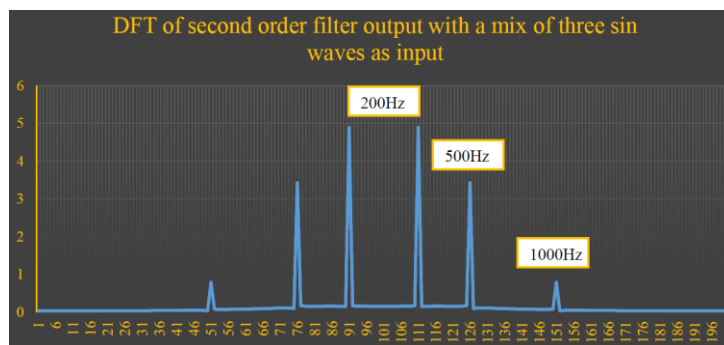


Fig. 13 The filtered signal in frequency domain.

Fig 12 shows the filter output response to a mixture of three sin waves input. it can be observed from the graph that the shape of the signal has been changed which means some frequencies have been blocked by the filter. However, it is not clear in the time domain to describe which frequency has been blocked specifically. To observe the signal properly and get the idea of what has been blocked by the filter and what has been allowed to pass, DFT of the output signal

wastaken. This illustrates the output signal in frequency domain. The 1000Hz frequency is more affected than 500Hz as it is not in the pass range.

As it is noticeable from Figure 13, the 200 Hz frequency signal has been allowed to pass without any attenuation in the amplitude as it is lower than the cut off frequency. The frequency that is at cut off frequency has limited influence. The 1000 Hz which is basically higher than the cut off frequency has been almost attenuated. Theoretically, as the order of the filter gets higher, the steeper of the frequency response curve of the filter gets. Accordingly, more attenuation is expected for the frequency 1000Hz with the 3rd order filter. It was noted that, the amplitude response of the filter at the cut off frequency is equal to 0.707 which is derived as follows:

$$M_{lp} = \frac{1}{\sqrt{1 + \left(\frac{1}{\left(\frac{f_{\text{equals to } f_c}}{f_c}\right)}\right)^{2n}}} = \frac{1}{\sqrt{1 + \left(\frac{1}{\left(\frac{f_c}{f_c}\right)}\right)^{2n}}} = 0.707$$

Therefore, regarding to the frequency at 500Hz, there will not be considerable change between the effect that the second or third order filter make.

3.2 IMPLEMENTATION OF THE THIRD ORDER BUTTERWORTH FILTER:

In this part, third order Butterworth filter which is designed to be low pass filter that has a steeper response than that the first and the second filters. It passes frequencies that are in the range from 0 to the cut off frequency. This third order Butterworth filter was designed with a cut off frequency of 500 Hz and a sampling frequency of 4000 Hz.

Similar to the procedure applied on the second order filter, An input signal (mixture of 3 sine wave) was passed through the 3rd order filter to examine its superiority and the output response of the filter is shown in figures 14 and 15.

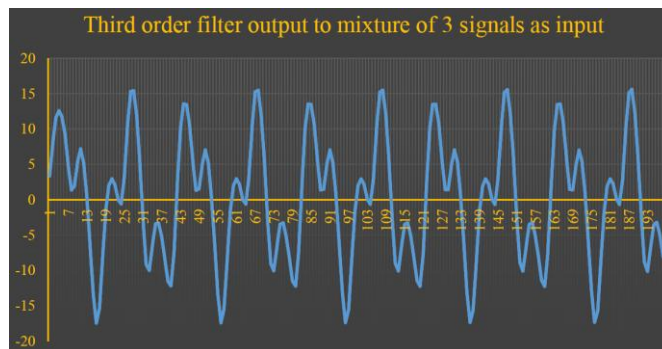


Fig.14 Output response of third order filter to mix of sin waves in time domain.

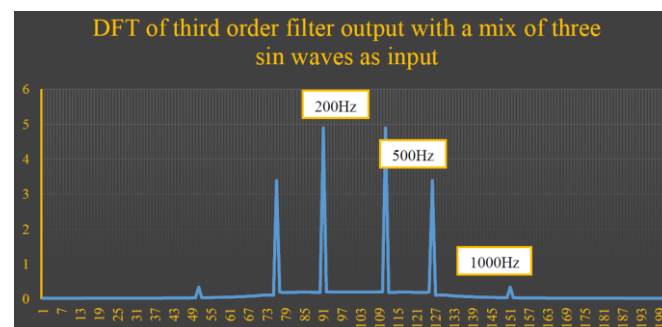


Fig. 15. Output response of third order filter to mix of sin waves in frequency domain.

It can be observed from the results that any frequency lower than the cut off frequency is allowed to pass and the frequency that is at cut off frequency has passed with a reduction in the amplitude. From figure 15, it is understandable that the effect of second order and the third order on the frequency 500Hz is similar which confirms what it has been mentioned early. Importantly, the main difference between this filter and the second order is the effect on the frequency that at 1000Hz. When the third order filter is applied, this frequency is almost blocked as it can be seen its amplitude is close to zero.



IV.CONCLUSION

In this paper, two different types of low pass Butterworth filter have been investigated. As it is known, in practical, there is no ideal filter. As a consequence of this, perfect results were not expected. However, it was obvious that the output response of the third order filter provided better response than the second order which the results demonstrate. Notably, the effect of the second order filter and third order filter on the frequency at 500 Hz are nearly similar which is expected, as this frequency equals to the cut off frequency that is used to design the filters. The amplitude response of low pass Butterworth filter when the frequency equals to cut off frequency is 0.707. In terms of the accuracy, it is obvious that the accuracy is associated with the flatness, in which the flatness of the output response increases as the order of the filter increases. This results that the frequencies that are higher than cut off frequency will be more affected or eliminated.

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